Effect of Polarization on Spacecraft Radiation Heat Transfer

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The fact that polarization affects radiation heat transfer is pointed out and illustrated with calculations for the interreflection of solar energy among spacecraft components. The general case of radiation transfer in an enclosure with account taken of polarization is formulated, and special cases are set up in detail for engineering calculations. These special cases are specular reflection from isotropic and anisotropic solids and one reflection of solar energy from an imperfectly diffuse solid. Qualitative rules are given to guide the engineer in deciding whether or not polarization should be considered.

Nomenclature

a = Fresnel quantity, Eq. (29)
A = area, cm²
b = Fresnel quantity, Eq. (30)
c = velocity of light, μ/sec
d = differential operator
$[D]$ = direction matrix, Eqs. (36) and (37)
$[E]$ = energy matrix, Eq. (45)
F = shape factor
G = irradiation, w-cm^{-2}-μ^{-1}
$\mathbf{i}$ = unit vector in x direction
i = unit imaginary number, $-1^{1/2}$
I = radiant intensity, w-cm^{-2}-μ^{-1}-sr^{-1}
$\mathbf{j}$ = unit vector in y direction
$\mathbf{k}$ = unit vector in z direction
k = absorptive index, Eq. (5)
M = magnitude of Z
n = refractive index, Eq. (5)
$\mathbf{p}$ = unit vector parallel to plane of incidence
$\mathbf{p}'$ = unit vector parallel to plane of emergence
q = heat flux, w-cm^{-2}
r = Fresnel quantity, Eqs. (2) and (3)
R = distance along path of ray
$\mathbf{R}$ = unit vector along path of ray
$\mathbf{s}$ = unit vector perpendicular to plane of incidence
$\mathbf{s}'$ = unit vector perpendicular to plane of emergence
$[X]$ = polarization matrix
Z = complex field strength, Eqs. (14) and (15)
$\mathcal{I}$ = imaginary part of a complex number
$\mathcal{R}$ = real part of a complex number
$\mathcal{E}$ = electric field strength multiplied by factor $(c/4\pi)^{1/2}$
α = absorptivity
β = angle, Eqs. (25) and (26), rad
δ = phase angle, rad
θ = polar angle of incidence, rad
θ' = polar angle of emergence
λ = wavelength, μ
ν = frequency, cps
π = 3.1416
ρ = reflectivity
φ = azimuthal angle of incidence, rad
φ' = azimuthal angle of emergence, rad
Ω = solid angle, sr

Subscripts

A = anisotropic
b = blackbody
B = bidirectional
D = perfectly diffuse
i = surface number
I = isotropic solid
j = surface number
M = specularly reflecting solid
n = surface number
p = direction parallel to plane of incidence
p' = direction parallel to plane of emergence
r = refracted ray
s = direction perpendicular to plane of incidence
s' = direction perpendicular to plane of emergence
S = from the sun
u = third Stokes coefficient
v = fourth Stokes coefficient
1 = surface 1
2 = surface 2
3 = surface 3

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I. Introduction

FIRST-GENERATION spacecraft had simple convex exteriors. Interiors were usually painted or anodized metals with relatively low reflectivities. The geometry in the first instance and the properties in the second precluded significant interreflections in the process of radiation heat transfer. The present generation of spacecraft is marked by complex configurations, some high-reflectivity surfaces, and, consequently, multiple reflections. Since radiation heat transfer is an important factor in determining spacecraft operating temperatures, methods are needed to compute transfer rates with proper account taken of interreflections.

Engineering calculations[1] of radiant heat transfer often employ all nine of the following assumptions:

1) Surface elements are plane and much larger in extent than the wavelength of the radiation. The elements are separated by distances that are large compared to their lateral dimensions and, therefore, large compared to the wavelength, i.e., diffraction can be neglected.

2) The body is so close to a state of thermodynamic equilibrium (defined by temperature T) that its emission, absorption, reflection, and transmission characteristics are those that may be measured or calculated at the equilibrium state. Fluorescence is neglected.

3) The intensity of radiation I is constant in a nonabsorbing, nonscattering medium along the line of sight.

4) The irradiation on element 1 from element 2 produces a reflected and transmitted intensity and a heating rate that is additive to that produced by irradiation on element 1 by any other element 3.

5) The magnitudes of the heating rate and reflected and transmitted radiation are fixed fractions of the irradiation independent of the state of polarization of the irradiation.

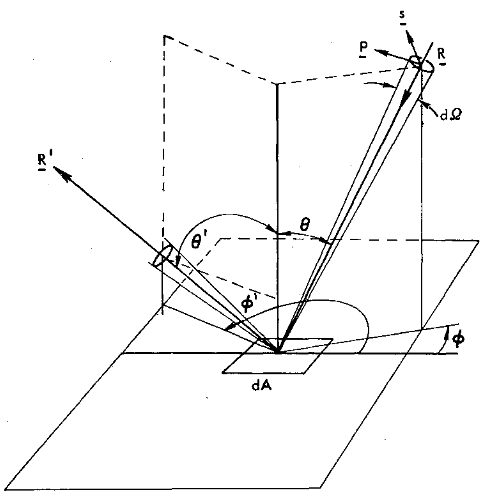

Fig. 1 Angles of incidence and emergence.

6) The fractions of the irradiation absorbed, transmitted, and reflected are not functions of the direction of the incident radiation, i.e., the surfaces emit diffusely.

7) The intensity of radiation from a surface element is not a function of direction, i.e., the surfaces reflect diffusely.

8) The intensity is independent of location on a set of finite surfaces.

9) The fractions of irradiation absorbed, transmitted, and reflected are not functions of wavelength, i.e., the surfaces are gray.

In the past few years, much progress has been made in finding methods for the engineer to avoid the last three restrictions when necessary. Bevans and Dunkle[2] showed how calculations using the first eight assumptions can be made by using spectral characteristics and approximating the needed integration by a summation over spectral bands. Assumption 8 is usually employed to approximate the integral formulation of radiant transfer with a set of algebraic equations and/or an equivalent electrical network.[3] For very simple geometries, it is possible to solve the integral equation approximately and, in two cases, even exactly. Sparrow et al.[4] have recently reported such results. A recent and quite significant advance by Sparrow and Eckert[5, 6] has been the lifting of assumption 7 for the case of plane specular surfaces. Edwards[7] has shown the ease with which assumption 6 can be eliminated for the convex space vehicle. Bevans and Edwards[8] have also explored some approximations that can be used to avoid assumptions 6 and 7 when interreflections occur.

One factor that has not been considered in engineering heat-transfer calculations is polarization. Although thermal radiation direct from the sun may be treated as natural or unpolarized, oblique reflection of this radiation may polarize it, and the amount of energy absorbed by a body irradiated obliquely depends upon the state of polarization of the incident radiation.

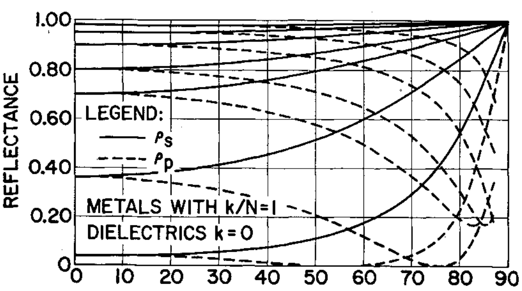

Fig. 2 Reflectivity components for isotropic plane surfaces.

The radiant intensity I (power per unit of normal area, solid angle, and spectral bandwidth) is used to formulate radiation transfer. For example, the solar irradiation $G_S = \int I d\Omega$, power per unit of normal receiver area and spectral bandwidth, is essentially the product of the solar intensity and small solid angle subtended by the sun. The intensity I is proportional to the time and spectral average of the square of the electric field vector $\mathcal{E}$, which oscillates perpendicular to the direction of propagation. This vector may be resolved into mutually perpendicular components, $\mathcal{E}_p$ and $\mathcal{E}_s$, which oscillate in the p and s directions.[9] It is most convenient to take p parallel and s perpendicular, respectively, to the plane of incidence or emergence containing the propagating ray and the surface normal. If there is a fixed phase relation between the two components, the radiation is said to be coherent. The average square of an electric field component $\mathcal{E}_p$ is proportional to the intensity component I_p. The sum of I_p and I_s equals I. Natural or unpolarized radiation has equal components I_p and I_s and no fixed phase relation between $\mathcal{E}_p$ and $\mathcal{E}_s$.

The purpose of the present paper is to show how to calculate radiant transfer without employing assumption 5. In Sec. II, the simple case of reflection of solar radiation from isotropic surfaces is considered. This case is taken first for two reasons: 1) to set up a technically important but simple case for practical engineering calculations, and 2) to illustrate the nature of the effect of polarization with numerical examples. In Sec. III, the more general case of specular reflection from anisotropic bodies (such as a solar cell with cemented cover glass) is presented. In the Appendix, the general problem is formulated in order to complete the presentation, even though immediate application of such a general formulation in engineering is not foreseen. A reduction of the general problem to a special case amenable to engineering application is made at the end of the Appendix. Circumstances under whicn polarization should not be neglected are summarized in Sec. IV.

II. Reflection of Solar Radiation from Isotropic Planes

A. Coincidence of Principal Directions

Reflection from an isotropic body with a plane surface is specular and is described by the Fresnel relations.[10] When solar radiation, which is assumed to be natural or incoherent, reflects from such a surface, the fraction of the solar intensity

Table 1 Sample radiation characteristics

Surface	$\theta = 75°$			$\theta = 0°$
	ρ_p	ρ_s	ρ	ρ
Silicon solar cell with 3-mil cover glass, antireflection film, and ultraviolet reflection film (measured at 0.81 μ)	0.070	0.215	0.142	0.075
Evaporated aluminum 500 Å thick on glass (measured at 0.81 μ)	0.67	0.94	0.805	0.85
Black enamel paint, two coats on one coat of zinc chromate primer on aluminum (measured at 0.81 μ)	0.105	0.32	0.21	0.045
Dielectric with 1.5 refractive index	0.107	0.399	0.253	0.040
Metal with 1.75 refractive index and 8.5 absorptive index	0.739	0.976	0.858	0.912

polarized parallel to the plane of incidence is ρ_p, whereas that polarized perpendicular to the plane of incidence is ρ_s. The magnitudes depend upon the angle of incidence θ (Figs. 1 and 2) and the refractive and absorptive indices n and k:

$$\rho_p(\theta) = \mathbf{r}_p\mathbf{r}_p^* \quad (1)$$

$$\mathbf{r}_p = (\cos\boldsymbol{\theta}_r - \mathbf{n}\cos\theta)/(\cos\boldsymbol{\theta}_r + \mathbf{n}\cos\theta) \quad (2)$$

$$\rho_s(\theta) = \mathbf{r}_s\mathbf{r}_s^* \quad (3)$$

$$\mathbf{r}_s = (\cos\theta - \mathbf{n}\cos\boldsymbol{\theta}_r)/(\cos\theta + \mathbf{n}\cos\boldsymbol{\theta}_r) \quad (4)$$

$$\mathbf{n} = n - ik \quad (5)$$

$$\mathbf{n}\sin\boldsymbol{\theta}_r = \sin\theta \quad (6)$$

Variation of ρ_p and ρ_s with θ is shown in Fig. 2 for various values of n and k.

If solar radiation reflects from one such surface and is incident on another so that the p and s directions fixed by the first surface coincide with those of the second (Fig. 3), the rate of heating by the reflected solar radiation is

$$q_{S-1-2} = \int_0^\infty [\alpha_2\rho_1]G_S\cos\theta_2 d\lambda$$

$$q_{S-1-2} = \int_0^\infty [\alpha_{2p}(\theta_2)\rho_{1p}(\theta_1) + \alpha_{2s}(\theta_2)\rho_{1s}(\theta_1)]\left(\frac{G_S}{2}\right)\cos\theta_2 d\lambda \quad (7)$$

where for opaque bodies

$$\alpha_{2p}(\theta_2) = 1 - \rho_{2p}(\theta_2) \text{ etc.} \quad (8)$$

and G_S is the solar irradiation per unit spectral bandwidth on a surface normal to the sun's rays. The quantity $\frac{1}{2}G_S\cos\theta_2$ is the solar irradiation per unit area of surface 2, which is s- or p-polarized. The term $\rho_{1p}(\theta_1)$ gives the fraction of this irradiation which is reflected by surface 1 to surface 2, and $\alpha_{2p}(\theta_2)$ gives the fraction of this irradiation which is not reflected and, therefore, absorbed by surface 2.

This case may be used to investigate whether or not polarization is of any technical importance. Consider Fig. 3 with $\theta_1 = \theta_2 = 75°$ and with the characteristics of surfaces 1 and 2 shown in Table 1. The data in Table 1 are from measurements made with an integrating sphere[11] and polarizing prism or from calculations made using the Fresnel relations, Eqs. (1–6). The exact value of $\alpha_2\rho_1$ is given by Eq. (8). Standard engineering practice is to ignore polarization effects and use directional values

$$[\alpha_2\rho_1] \simeq \alpha_2(\theta_2)\rho_1(\theta_1) \quad (9)$$

or even normal values

$$[\alpha_2\rho_1] \simeq \alpha_2(0)\,\rho_1(0) \quad (10)$$

Table 2 shows these approximations to be quite poor. Errors of +92 to −81% are seen to result.

If two reflections occur before the radiation is incident on a surface of interest and all of the principal directions coincide, then the term representing absorption of p-polarized radiation contains two reflectivities for the p directions.

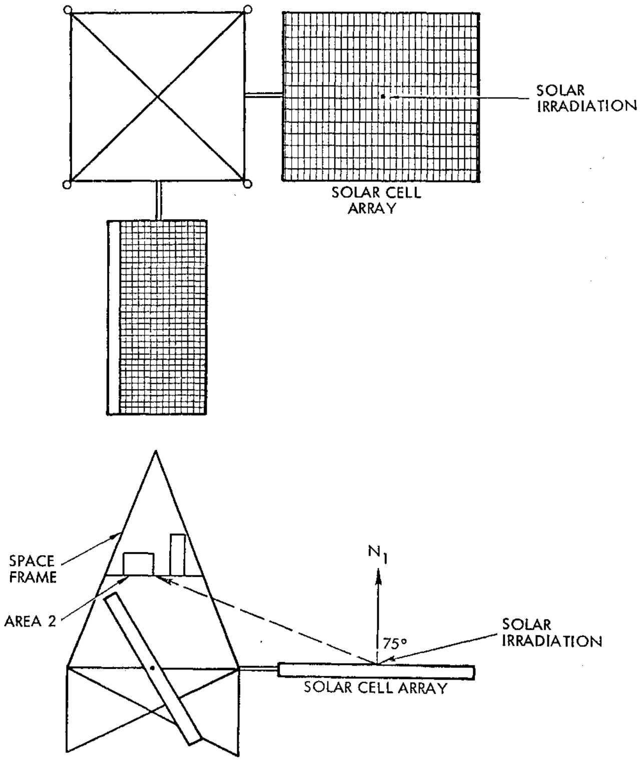

Fig. 3 Example of once-reflected solar radiation.

B. Surfaces Askew

When the radiation reflected from an isotropic plane is incident on a second plane at an angle such that the p_1' direction of emergence from the first does not coincide with the p_2 direction for incidence on the second, it is necessary to resolve the electric vector into new components in the p_2 and s_2 directions. For the plane isotropic surface, the heat rate absorbed is

$$q_2 = \int_0^\infty \int_\Omega (\alpha_{2p_2}I_{p_2} + \alpha_{2s_2}I_{s_2})\cos\theta_2 d\Omega d\lambda \quad (11)$$

The intensity directed toward surface 2 after reflection from surface 1 is

$$I_{p_1}' = \rho_{1p_1}'(I_S/2) \quad (12)$$

$$I_{s_1}' = \rho_{1s_1}'(I_S/2) \quad (13)$$

where I_S is the intensity of solar radiation that is assumed to be natural radiation. In order to convert I_{p_1}' and I_{s_1}' to I_{p_2} and I_{s_2}, it is necessary to consider the phase change upon reflection.

It is the electric vector $\mathcal{E}$ that can be resolved into a new set of mutually perpendicular components. The components of vector $\mathcal{E}$ are most conveniently represented in terms of complex numbers:

$$Z_p = M_p\exp\{i[2\pi\nu(t - R/c) + \delta_p]\} \quad (14)$$

$$Z_s = M_s\exp\{i[2\pi\nu(t - R/c) + \delta_s]\} \quad (15)$$

Table 2 Sample calculation of the effect of polarization on the absorption of once-reflected solar radiation[a]

System			Ratio of approximate to exact solution	
Reflector	Absorber	Exact solution $[\alpha_2\rho_1]$, Eq. (7)	Directional values, Eq. (9)	Normal values, Eq. (10)
Solar cell	Aluminum	0.0180	1.54	0.63
Solar cell	Black paint	0.1045	1.07	0.68
Dielectric,[b] $n = 1.5$	Metal,[c] $n = 1.75, k = 8.5$	0.0187	1.92	0.19

[a] $\theta_1 = \theta_2 = 75°$, coincidence of principal directions radiation characteristics from Table 1.
[b] For example, glass.
[c] For example, aluminum at 0.95 μ (Ref. 12).

where

$$\mathcal{E}_p = \Re Z_p \tag{16}$$

$$\mathcal{E}_s = \Re Z_s \tag{17}$$

and where R is the distance along the line or propagation, t is time, ν is frequency, c is speed of propagation, δ is the instantaneous phase angle, $\Re$ denotes the real part of the complex number, $\Im$ denotes the imaginary part, and i is the imaginary unit number $-1^{1/2}$. Wavelength is related to frequency through

$$\nu = c/\lambda \tag{18}$$

Resolving $\mathcal{E}$ into new components results in

$$Z_{p_2} = \mathbf{p}_1'\cdot\mathbf{p}_2\, Z_{p_1'} + \mathbf{s}_1'\cdot\mathbf{p}_2\, Z_{s_1'} \tag{19}$$

$$Z_{s_2} = \mathbf{p}_1'\cdot\mathbf{s}_2\, Z_{p_1'} + \mathbf{s}_1'\cdot\mathbf{s}_2\, Z_{s_1'} \tag{20}$$

If the constant of proportionality between I and $\mathcal{E}^2$ is absorbed into the magnitude M, there results

$$I_{p_1'} = \langle M_{p_1'}{}^2\rangle = \langle Z_{p_1'}Z_{p_1'}{}^*\rangle \qquad I_{p_2} = \langle Z_{p_2}Z_{p_2}{}^*\rangle \tag{21}$$

$$Z_{s_2} = \langle M_{s_1'}{}^2\rangle = \langle Z_{s_1'}Z_{s_1'}{}^*\rangle \qquad I_{s_2} = \langle Z_{s_2}Z_{s_2}{}^*\rangle \tag{22}$$

where the asterisk denotes the complex conjugate, and the angular brackets denote a time and wavelength average over a short time period and narrow wavelength band around time t and wavelength λ.

The new set of intensity components is then obtained by substituting Eqs. (19) and (20) into Eqs. (21) and (22):

$$\left.\begin{aligned} I_{p_2} &= (\mathbf{p}_1'\cdot\mathbf{p}_2)^2\langle Z_{p_1'}Z_{p_1'}{}^*\rangle + (\mathbf{s}_1'\cdot\mathbf{p}_2)^2\langle Z_{s_1'}\cdot Z_{s_1'}{}^*\rangle + \\ &\qquad (\mathbf{p}_1'\cdot\mathbf{p}_2)(\mathbf{s}_1'\cdot\mathbf{p}_2)\langle[Z_{p_1'}Z_{s_1'}{}^* + Z_{p_1'}{}^*Z_{s_1'}]\rangle \\ I_{p_2} &= \cos^2\beta_{1-2}I_{p_1'} + \sin^2\beta_{1-2}I_{s_1'} + \\ &\qquad \tfrac{1}{2}(\sin 2\beta_{1-2})\langle[2M_{p_1'}M_{s_1'}\cos(\delta_{p_1'} - \delta_{s'_1})]\rangle \end{aligned}\right\} \tag{23}$$

$$\left.\begin{aligned} I_{s_2} &= (\mathbf{p}_1'\cdot\mathbf{s}_2)^2\langle Z_{p_1'}Z_{p_1'}{}^*\rangle + (\mathbf{s}_1'\cdot\mathbf{s}_2)^2\langle Z_{s_1'}Z_{s_1'}{}^*\rangle + \\ &\qquad (\mathbf{p}_1'\cdot\mathbf{s}_2)(\mathbf{s}_1'\cdot\mathbf{s}_2)\langle[Z_{p_1'}Z_{s_1'}{}^* + Z_{p_1'}{}^*Z_{s_1'}]\rangle \\ I_{s_2} &= \sin^2\beta_{1-2}I_{p_1'} + \cos^2\beta_{1-2}I_{s_1'} - \\ &\qquad \tfrac{1}{2}(\sin 2\beta_{1-2})\langle[2M_{p_1'}M_{s_1'}\cos(\delta_{p_1'} - \delta_{s_1'})]\rangle \end{aligned}\right\} \tag{24}$$

where

$$\mathbf{p}_1'\cdot\mathbf{p}_2 = \cos\beta_{1-2} \tag{25}$$

$$\mathbf{p}_1'\cdot\mathbf{s}_2 = -\sin\beta_{1-2} \tag{26}$$

It is seen that when the surfaces are askew it is necessary to know the average cosine of the phase difference $(\delta_{p_1'} - \delta_{s_1'})$. The phase difference arises from reflection from surface 1 and prior reflections. The Fresnel relations[10] give the phase difference occurring upon reflection implicity:

$$\tan\delta_{1p} = \frac{2b_1\cos\theta(a_1{}^2 + b_1{}^2 - \sin^2\theta_1)}{a_1{}^2 + b_1{}^2 - (n_1{}^2 + k_1{}^2)\cos^2\theta_1} \tag{27}$$

$$\tan\delta_{1s} = \frac{2b_1\cos\theta}{a_1{}^2 + b_1{}^2 - \cos^2\theta_1} \tag{28}$$

where

$$a^2 = \tfrac{1}{2}\{[(n^2 - k^2 - \sin^2\theta)^2 + 4n^2k^2]^{1/2} + (n^2 - k^2) - \sin^2\theta\} \tag{29}$$

$$b^2 = \tfrac{1}{2}\{[(n^2 - k^2 - \sin^2\theta)^2 + 4n^2k^2]^{1/2} - (n^2 - k^2) + \sin^2\theta\} \tag{30}$$

The magnitudes and phase relations of the reflected radiation from an isotropic surface are

$$Z_{p_1'} = (\rho_{1p})^{1/2}M_{p_1}\exp\{i[2\pi\nu(t - R/c) + \delta_{p_1} + \delta_{1p}]\} = \rho_{1p}{}^{1/2}\exp(i\delta_{1p})Z_{p_1} \tag{31}$$

$$Z_{s_1'} = \rho_{1s}{}^{1/2}\exp(i\delta_{1s})Z_{s_1} \tag{32}$$

These quantities may be used to find the quantity appearing in the last term of Eq. (23) or (24). This quantity is the third Stokes coefficient[9] and is designated by

$$I_{u_1'} = 2M_{p_1'}M_{s_1'}\cos(\delta_{p_1'} - \delta_{s_1'}) = \langle\Re(2Z_{p_1'}Z_{s_1'}{}^*)\rangle \tag{33}$$

If the irradiation on surface 1 is incoherent, the phase angles δ_{p_1} and δ_{s_1} change randomly with time and wavelength, so that the average cosine is zero, i.e., I_{u_1} and $I_{u_1'}$ are both zero. In this case of one reflection, the rate of heating per unit area on surface 2 is from Eqs. (11, 23, 24, 31, and 32) and the definition of G_S:

$$\begin{aligned} q_{S-1-2} &= \int_0^\infty [\alpha_2\rho_1]\cos\theta_2 G_S d\lambda = \\ &\int_0^\infty \{\alpha_{2p}(\theta_2)\rho_{1p}(\theta_1)\cos^2\beta_{1-2} + \\ &\alpha_{2p}(\theta_2)\rho_{1s}(\theta_1)\sin^2\beta_{1-2} + \alpha_{2s}(\theta_2)\rho_{1p}(\theta_1)\sin^2\beta_{1-2} - \\ &\alpha_{2s}(\theta_2)\rho_{1s}(\theta_1)\cos^2\beta_{1-2}\}G_S\left(\frac{\cos\theta_2}{2}\right)d\lambda \end{aligned} \tag{34}$$

It is seen that the phase change upon reflection from surface 1 has no effect upon the heating rate.

However, if solar radiation is reflected from surface 1 to surface 2 and then reflected to surface 3, the phase change introduced by surface 2 is of consequence. The procedure just set forth shows how to express Z_{p_2} and Z_{s_2} in terms of $Z_{p_1'}$ and $Z_{s_1'}$ by Eqs. (19) and (20). The quantities $Z_{p_1'}$ and $Z_{s_1'}$ are given in terms of Z_{p_1} and Z_{s_1} by Eqs. (31) and (32). The I_{p_3} and I_{s_3} intensities are given by equations similar to Eqs. (23) and (24) and the heat rate by an equation similar to Eq. (11), where the subscript 2 is replaced by 3. This procedure may be extended to any number of surfaces. However, the matrix notation presented below leads to a simpler formulation.

C. Matrix Representation

The intensity may be represented by a four-element column matrix or vector[9]

$$[I] = \begin{bmatrix} I_p \\ I_s \\ I_u \\ I_v \end{bmatrix} = \begin{bmatrix} Z_pZ_p{}^* \\ Z_sZ_s{}^* \\ \Re(2Z_pZ_s{}^*) \\ \Im(2Z_pZ_s{}^*) \end{bmatrix} \tag{35}$$

where $\Re$ denotes the real part and $\Im$ the imaginary part. Equations (34) and (35) may be used to transform from the p_1', s_1' to the p_2, s_2 direction, as was done to derive Eqs. (23) and (24):

$$\begin{bmatrix} I_{p_2} \\ I_{s_2} \\ I_{u_2} \\ I_{v_2} \end{bmatrix} = \begin{bmatrix} \cos^2\beta_{1-2} & \sin^2\beta_{1-2} & \frac{1}{2}\sin 2\beta_{1-2} & 0 \\ \sin^2\beta_{1-2} & \cos^2\beta_{1-2} & -\frac{1}{2}\sin 2\beta_{1-2} & 0 \\ -\sin 2\beta_{1-2} & \sin 2\beta_{1-2} & \cos 2\beta_{1-2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{p_1'} \\ I_{s_1'} \\ I_{u'_1} \\ I_{v_1'} \end{bmatrix} \tag{36}$$

$$[I_2] = [D_{1-2}]\cdot[I_1] \tag{37}$$

The reflectivity of a mirror-like isotropic plane represented by Eqs. (31) and (32) is, in matrix notation,

$$[\rho_{MI}] = \begin{bmatrix} \rho_{p'p} & 0 & 0 & 0 \\ 0 & \rho_{s's} & 0 & 0 \\ 0 & 0 & \rho_{u'u} & \rho_{v'u} \\ 0 & 0 & \rho_{u'v} & \rho_{v'v} \end{bmatrix} \tag{38}$$

where

$$\rho_{p'p} = \rho_p \tag{39}$$

$$\rho_{s's} = \rho_s \tag{40}$$

$$\rho_{u'u} = \rho_{v'v} = (\rho_p\rho_s)^{1/2}\cos(\delta_p - \delta_s) \tag{41}$$

$$\rho_{u'v} = -\rho_{v'u} = (\rho_p\rho_s)^{1/2}\sin(\delta_p - \delta_s) \tag{42}$$

The reflected intensity matrix is then

$$[I'] = [\rho_{MI}]\cdot[I] \tag{43}$$

The intensity $I = I_p + I_S$ is given in matrix notation by the operator $[E]^T$ on matrix $[I]$:

$$I = [E]^T\cdot[I] \tag{44}$$

where $[E]^T$ is the transpose of $[E]$, that is,

$$[E]^T = [1 \quad 1 \quad 0 \quad 0] \qquad [E] = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{45}$$

The specular directional reflectivity vector is defined such that the reflected intensity I' is given by

$$I' = [\rho]^T\cdot[I] \tag{46}$$

Equations (43–45) indicate that

$$[\rho_I]^T = [E]^T\cdot[\rho_{MI}] \tag{47}$$

For the plane isotropic solid,

$$[\rho_I] = \begin{bmatrix} \rho_s \\ \rho_p \\ 0 \\ 0 \end{bmatrix} \tag{48}$$

Similarly, the portion of the intensity I which is absorbed is $[\alpha]^T\cdot[I]$, where for the isotropic solid the matrix absorptivity is

$$[\alpha_I] = \begin{bmatrix} 1-\rho_s \\ 1-\rho_p \\ 0 \\ 0 \end{bmatrix} \tag{49}$$

In matrix notation, the solar irradiation reflected from surface 1 to 2 is

$$q_{S-1-2} = \int_0^\infty [\alpha_2\rho_1]\cos\theta_2 G_S d\lambda = \int_0^\infty [\alpha_2]^T\cdot[D_{1-2}]\cdot[\rho_1]\cdot[X_b]G_S\cos\theta_2 d\lambda \tag{50}$$

where $[X_b]$ is the column matrix denoting unit incoherent or blackbody intensity:

$$[X_b] = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \end{bmatrix} \tag{51}$$

This notation has the advantage that any number of reflections can be accommodated easily:

$$q_{S-1-\ldots-n} = \int_0^\infty [\alpha_n]^T\cdot[D_{(n-1)-n}]\cdot[\rho_{M(n-1)}]\ldots [D_{1-2}]\cdot[\rho_{MI}]\cdot[X_b]G_S\cos\theta_n d\lambda \tag{52}$$

D. Description of Geometry

In order to employ Eq. (52), the matrices D_{i-j} are needed. The matrix elements are defined by Eqs. (37, 25, and 26) in terms of the unit vectors $\mathbf{p}_i'$, $\mathbf{s}_i'$, $\mathbf{p}_j$, $\mathbf{s}_j$, etc., parallel and perpendicular to the planes of incidence and emergence and mutually perpendicular to the direction of propagation. It is also necessary to determine which portions of the areas are illuminated by direct or reflected solar radiation. Consideration is given here to the former problem of determining the direction cosines of the unit vectors. The latter problem of shadowing may be solved by modeling, graphical descriptive geometry, or analytic geometry and is not considered in this paper.

The direction of incidence is given by vector $\mathbf{R}_i$ and the surface normals by $\mathbf{N}_i$ and $\mathbf{N}_j$. The unit vector $\mathbf{s}_i$ is then

$$\mathbf{s}_i = (\mathbf{R}_i \times \mathbf{N}_i)/|\mathbf{R}_i \times \mathbf{N}_i| \tag{53}$$

where the vertical bars denote the magnitude. Unit vector $\mathbf{p}_i$ is

$$\mathbf{p}_i = \mathbf{s}_i \times \mathbf{R}_i \tag{54}$$

The angle of incidence is

$$\cos\theta_i = -\mathbf{N}_i\cdot\mathbf{R}_i \tag{55}$$

Radiation emerges along direction $\mathbf{R}_i'$ given by

$$\mathbf{R}_i' = -(\mathbf{R}_i\cdot\mathbf{N}_i)\mathbf{N}_i + [\mathbf{R}_i\cdot(\mathbf{N}_i \times \mathbf{s}_i)](\mathbf{N}_i \times \mathbf{s}_i) \tag{56}$$

Principal directions of emergence are then

$$\mathbf{s}_i' = \mathbf{s}_i \tag{57}$$

$$\mathbf{p}_i' = \mathbf{s}_i' \times \mathbf{R}_i' \tag{58}$$

and the direction of incidence of the reflected radiation from surface i to surface j is

$$\mathbf{R}_j = \mathbf{R}_i' \tag{59}$$

Directions $\mathbf{s}_j$ and $\mathbf{p}_j$ are then found as in Eqs. (53) and (54).

With Eqs. (53–59), all of the desired relations can be found in terms of the direction cosines of the surface normals and the original line of incidence by carrying out the vector cross and dot multiplications.

E. Numerical Example for Two Reflections

Figure 4 shows a geometry in which solar radiation is reflected twice before it irradiates a surface of interest. Shown are the **i, j, k** vectors, the $\mathbf{N}_1$, $\mathbf{N}_2$, $\mathbf{N}_3$ surface normals, and $\mathbf{R}_1$, the direction of incident solar radiation. In this example, the principal directions do not coincide for all three surfaces. This system corresponds approximately to solar radiation striking a solar cell array, reflecting to a metal component, and being absorbed by a metal or glossy-black painted surface.

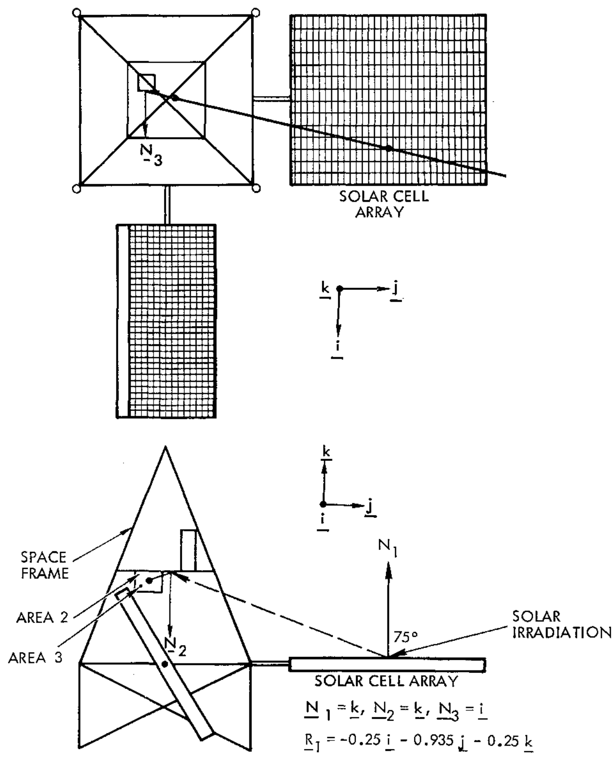

Fig. 4 Example of twice-reflected solar radiation.

Table 3 Sample calculation of the effect of polarization on the absorption of twice-reflected solar radiation[a]

System			Exact solution, Eq. (52),	Ratio of approximate to exact solution	
First reflector	Second reflector	Absorber	$q_{S\text{-}1\text{-}2\text{-}3}/G_S \cos\theta_3$	Directional values	Normal values
Dielectric, $n = 1.5$	Metal, $n = 1.75$, $k = 8.5$	Metal, $n = 1.75$, $k = 8.5$	0.0541	0.60	0.06
Dielectric, $n = 1.5$	Metal, $n = 1.75$, $k = 8.5$	Dielectric, $n = 1.5$	0.202	0.82	0.17

[a] Geometry per Fig. 4, $\theta_3 = 75.5°$.

Equations (53–59) are used to find the trigonometric quantities needed in Eq. (52). The radiation characteristics are calculated using the Fresnel equations for the isotropic plane solid. Results of sample calculations are shown in Table 3.

When the radiation characteristics for normal incidence are employed for this case, answers 6 to 16 times too small are obtained. If directional values are used, answers 18 to 40% too small are obtained.

III. Specular Reflection from Antisotropic Planes

Specular or mirror-like reflection can also occur from anisotropic or inhomogeneous bodies, for example, a solar cell covered with a layer of cement holding a cover glass with thin coatings. It is, in general, possible for the four components of the intensity matrix to interact within such a body, so that specular reflection from an anisotropic body with a laminated structure is described by a matrix with all 16 terms present:

$$[\rho_{MA}] = \begin{bmatrix} \rho_{p'p} & \rho_{p's} & \rho_{p'u} & \rho_{p'v} \\ \rho_{s'p} & \rho_{s's} & \rho_{s'u} & \rho_{s'v} \\ \rho_{u'p} & \rho_{u's} & \rho_{u'u} & \rho_{u'v} \\ \rho_{v'p} & \rho_{v's} & \rho_{v'u} & \rho_{v'v} \end{bmatrix} \tag{60}$$

Additional terms in the reflectivity matrix introduce additional terms in the absorptivity matrix. The directional reflectivity matrix is, as before,

$$[\rho]^T = [E]^T \cdot [\rho_{MA}] \tag{61}$$

and for an opaque surface

$$[\alpha] + [\rho] = [E] \tag{62}$$

so that

$$[\alpha] = \begin{bmatrix} \alpha_p \\ \alpha_s \\ \alpha_u \\ \alpha_v \end{bmatrix} = \begin{bmatrix} 1 - \rho_p \\ 1 - \rho_s \\ -\rho_u \\ -\rho_v \end{bmatrix} \tag{63}$$

Other than the preceding changes in the number and nature of the terms in the radiation property matrices, the formulation of the problem is identical to that for the isotropic solid.

IV. Summary and Conclusions

When heat-transfer calculations accounting for polarization are called for, the formulations presented here will facilitate them. Equation (52) gives the rate of solar heat absorbed by the nth surface after $n - 1$ reflections. The integral over wavelength in Eq. (52) is replaced by a summation for engineering calculations as shown in Ref. 2 or 7, for example. The properties of the isotropic solid are given by Eqs. (1–6, 27–30, 38–42, and 48–49). Geometrical quantities are given in Eqs. (25, 26, 37, and 53–59). Further cases are considered in the Appendix.

From the examples and formulations presented, it is possible to draw some qualitative conclusions as to when such calculations are called for. In the reflection of solar radiation from one solid isotropic surface to another, polarization should be considered when both reflector and absorber are irradiated at angles greater than approximately 60°, the reflector is a dielectric or semiconductor, and the absorber is a metal. The rate of absorption should, of course, be an important term in the total heat balance.

Errors from the neglect of polarization, but not directional variation, have been shown to indicate answers from approximately twice to one-half the correct answer for the examples presented. Neglect of polarization and directional variation yields an answer in error by more than a factor of 10 in one example presented. The engineer should be alert for the conditions leading to errors from neglect of polarization, just as he should be alert for those conditions making spectral and directional selectivity important.

Appendix: Reflection from Imperfectly Diffuse Materials

Reflection Distribution Matrix

A body such as a rough, porous, or transparent polycrystalline slab may reflect radiation into directions θ_2, φ_2 outside the direction for specular or mirror-like reflection $\theta_2 = \theta_1$, $\varphi_2 = \varphi_1 + \pi$. The directional distribution of the reflected radiation and its state of polarization for irradiation of a particular direction of incidence and a certain state of polarization are given by a bidirectional reflectivity matrix:

$$[I'(\theta', \varphi')] = [\rho_B(\theta, \varphi, \theta', \varphi')] \cdot [I(\theta, \varphi)] F dA \tag{A1}$$

where

$$FdA = (\cos\theta \cos\theta' / \pi R^2) dA \tag{A2}$$

A perfectly diffuse body has a bidirectional reflectivity matrix of

$$[\rho_{BD}] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{A3}$$

The bidirectional reflectivity or reflection distribution function is defined for incoherent directional irradiation and for the total energy.[13–15] Thus, it is defined by

$$I' = \rho_B I \, F_{1-2} dA_1 \tag{A4}$$

so that

$$\rho_B = [E]^T \cdot [\rho_B] \cdot [X_b] \tag{A5}$$

The quantity is defined so that a perfectly diffuse body has a bidirectional reflectivity of unity (it could be redefined so as to be $1/\pi$ or any arbitrary value):

$$\rho_{BD} = [1 \quad 1 \quad 0 \quad 0] \cdot \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \end{bmatrix} = 1 \tag{A6}$$

The directional reflectivity matrix is defined by summing the total radiation in all directions of emergence:

$$[\rho]^T = [\rho(\theta, \varphi)]^T = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\pi/2} [E]^T \cdot [\rho_B(\theta, \varphi, \theta', \varphi')] \sin\theta' \cos\theta' d\theta' d\varphi' \quad (A7)$$

The ordinary directional reflectivity for incoherent radiation is

$$\rho(\theta, \varphi) = [\rho(\theta, \varphi)]^T \cdot [X_b] \quad (A8)$$

For a mirror-like or specular surface $[\rho_B]$ contains delta functions, so that the right side of Eq. (A7) without the $[E]^T$ matrix multiplication becomes $[\rho_M]$. For an opaque surface, the absorptivity matrix is given by Eq. (63).

Integral Formulation

For an enclosure of opaque surfaces, the intensity is obtained from Eq. (A1) by integration over area and by the addition of emitted flux:

$$[I'] = \int_A [\rho_B] \cdot [I] F dA + [\alpha] \left(\frac{I_b}{2}\right) \quad (A9)$$

This notation represents four simultaneous integral equations. One method of approximate solution for the general case is to subdivide the area into a number of elements over which the intensity directed toward another element is assumed constant. It is not anticipated that Eq. (A9) or a set of simultaneous algebraic equations derived from it will come into use to treat the general case until computers become considerably faster and have more storage capability than at present. However, there are special simple cases that are tractable and technically important. One such case is shown below.

Special Case: One Reflection of Solar Radiation

Consider solar radiation that is reflected from one surface to another, as in Fig. 3. If the sun is far enough away, the irradiation may be considered unidirectional so that Eq. (A1) or (A9) gives the reflected intensity as

$$[I_1'] = [\rho_{B1}] \cdot [X_b] G_S \cos\theta_1 / \pi \quad (A10)$$

An element of surface 2 absorbs solar radiation from an element of surface 1 at the rate

$$dq_{S-1-2} = \int_0^\infty [\alpha_2]^T \cdot [I_1'] \frac{\cos\theta_1' \cos\theta_2}{R_{1-2}^2} dA_1 dA_2 \, d\lambda \quad (A11)$$

$$q_{S-1-2} = \int_0^\infty G_S \cos\theta \int_{A_1} \int_{A_2} [\alpha]^T \cdot [\rho_{B1}] \cdot [X_b] \frac{\cos\theta_1' \cos\theta_2}{\pi R_{1-2}^2} dA_1 dA_2 \quad (A12)$$

The geometric reflection-absorption factor occurring in Eq. (A12)

$$[\alpha_2 \rho_1 A_1 F_{1-2}] = \int_{A_1} \int_{A_2} [\alpha_2]^T \cdot [\rho_{B1}] \cdot [X_b] \frac{\cos\theta_1' \cos\theta_2}{\pi R_{1-2}^2} dA_1 dA_2 \quad (A13)$$

may be calculated by numerical means using calculated or measured values of the matrix elements in $[\alpha_2]$ and $[\rho_{B1}]$.

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